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# Reduction of Outage Probability in Fast Rayleigh Fading MIMO Channels Using OFDM

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**ABSTRACT**
Multiple-input multiple-output (MIMO) techniques are used in wireless communications for achieving high spectral efficiency; however, a fast fading spatial channel can increase the outage probability of a MIMO system if not taken care of. This paper investigates the use of orthogonal frequency division multiplexing (OFDM) modulation technique for a MIMO system operating in fast Rayleigh fading channels with the aim of eliminating outage probabilities in the MIMO systems. Simulation results show that the MIMO-OFDM system gives significant reduction in outage probabilities compared to the conventional MIMO system.
**Keywords**- Fast fading, outage probability, MIMO, OFDM, SNR, transmission rate, channel capacity

## I. Introduction

The rising demands for mobile wireless systems that are capable of providing very high data rates for multimedia applications with seamless quality of service (QoS) delivery have lead to the development of more robust technologies such as the multiple-input multiple-output (MIMO) communication systems. MIMO technology has been in existence for over a decade [1], [2]. A MIMO communication system exploits spatial diversity by utilizing multiple antennas at the transmitting and receiving ends of the system, which considerably increase the link capacity and reliability compared to the single-input single-output (SISO) communication systems [3]. The advancement in very large scale integration (VLSI) technology has made the implementation of MIMO antenna configuration possible in mobile devices, and MIMO has found applications in a number of standards such as UMTS, WLAN and WiMAX [4]. However, in highly-mobile scenarios, the MIMO channels experience rapid variations [5]; a phenomenon referred to as fast fading that could increase the probability of signal outages and results in performance degradation of the MIMO system.

A mobile wireless channel exhibits fast fading if the channel symbol period is greater than the coherence time of the channel or if the Doppler spread increases relative to the bandwidth of the transmitted signal [6], [7]; this can lead to outage of the signal or a complete loss of the signal.

Outage probability is the probability that the received SNR falls below a specified threshold value [8]. In a MIMO system, it is the probability that the average capacity of the system is below a specified rate. In other words, a system outage occurs when the instantaneous error probability cannot be made arbitrarily small with the information rate [9]. In wireless system designs, the typical outage probability target is 5% [8].

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation technique capable of mitigating ISI, and has been shown to be very effective for high-rate data transfer through dispersive channels [10], [11]. OFDM breaks a high-rate single carrier into parallel low-rate subcarriers [12], [13]. The subcarriers are given the minimum frequency separation required to maintain orthogonality between them in order to prevent intercarrier interference (ICI) [13]. OFDM modulation has found application in a number of digital wireless communication systems such as DAB, DVB and 3GPP LTE [14], [15].

A conventional MIMO system operating in a fast fading environment would require complex channel equalizers to be able to mitigate deep fades that may cause system outages; and the complexity of the required equalizers increases with the number of antennas of the MIMO system. Intensive researches are on going on the enhancement of MIMO systems as well as the suitability of MIMO-OFDM for providing very high QoS of broadband wireless communications [5], [16], [4], [12]; however, most of these investigations are based on bit error rate (BER) and spectral efficiency evaluations of the MIMO-OFDM. This work therefore shows how OFDM could be used to reduce outage probabilities of the MIMO system in fast fading environments.

## II. MIMO-OFDM System Model

The MIMO scheme considered in this study is the spatial multiplexing which offers very high spectral efficiency. A MIMO-OFDM system model of 2x2 ( $N_R$ x $N_T$ ) antenna configuration is shown in Fig. 1. The system consists of $N_T$ transmit and $N_R$ receive antennas, and assuming no channel state information at the transmitter (CSIT). The information bits to be transmitted are first demultiplexed into $N_T$ data streams; each bit stream is then mapped onto M-QAM constellation where M is the constellation size. The serial M-QAM stream at each transmitter branch is converted into K parallel streams and IFFT is applied to form time-domain signals. The parallel time-domain signals are converted back into a serial signal and a cyclic prefix (CP) is appended to form the OFDM signal. After filtering, each transmitter sends an OFDM signal over a fast Rayleigh fading MIMO channel; this implies non line of sight (NLoS) between the transmitter and the receiver. At the receiving end, the process is reversed to recover the information bits. The length of the CP is assumed to be longer than the channel delay spread to mitigate the fading effect. After the FFT demodulation, the received OFDM signal of the k-th subcarrier at the i-th receive antenna can be expressed as:

$$y_i^k(t) = \sum_{j=1}^{N_T} H_{i,j}^k(t) s_j^k(t) + z_i^k(t),$$

$$i = 1,2,..., N_R$$

(1)

where $H_{i,j}^k(t)$ is the channel coefficient between the j-th transmitter and the i-th receiver, $s_j^k(t)$ is the transmitted data symbol on k-th subcarrier, $z_i^k(t)$ is the additive white Gaussian noise (AWGN) at i-th receive antenna and t is the time instant. The MIMO channel matrix corresponding to the k-th subcarrier of an OFDM symbol is given by:

$$\hat{H}^k(t) = \begin{bmatrix} H_{1,1}^k(t) & H_{1,2}^k(t) & \cdots & H_{1,N_T}^k(t) \\ H_{2,1}^k(t) & H_{2,2}^k(t) & \cdots & H_{2,N_T}^k(t) \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}$$

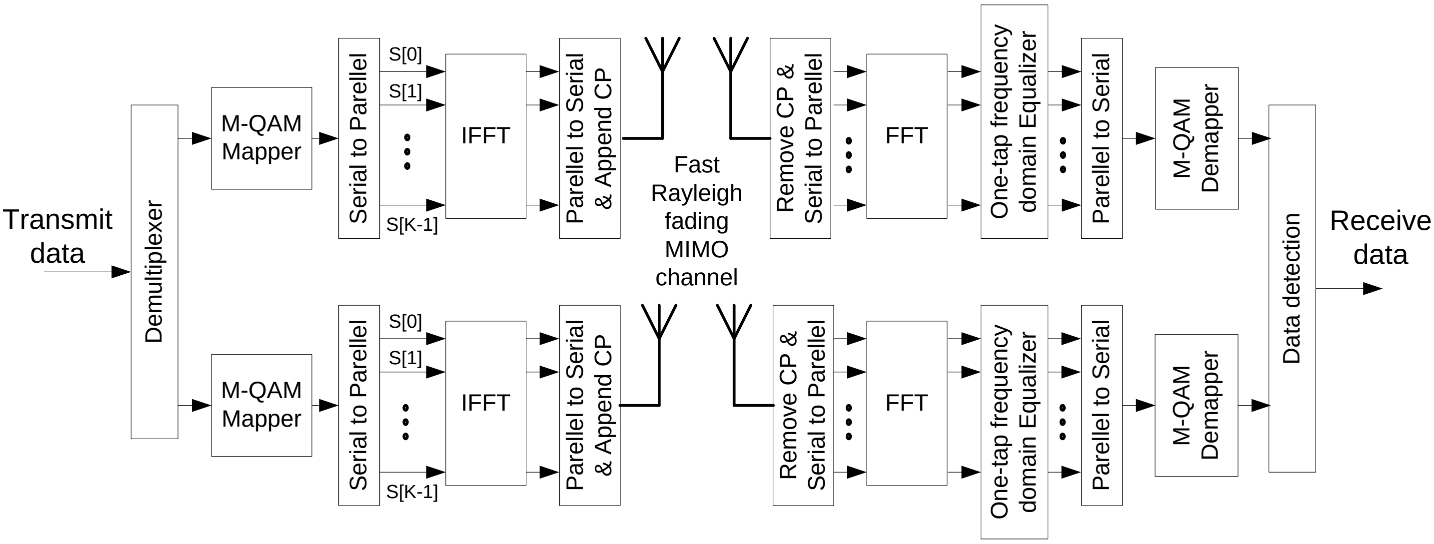


Fig. 1: 2x2 MIMO-OFDM System Model

The MIMO-OFDM channel coefficient is given as:

$$H^k(t) = \sum_{l}^{L-1} h_l(t) e^{-j2\pi f_m tk/T},$$

$$k = 0,1,\ldots,K-1$$

(3)

where $h_l(t)$ is the Rayleigh fading coefficient for the $l$-th path modelled as an independent zero mean random Gaussian process, $L$ is the number of resolvable multipath components, $T$ is the OFDM symbol period, $f_m$ is the maximum Doppler frequency shift caused by the relative motion between the transmitter and the receiver. $f_m$ is a function of the speed of the mobile station and the carrier frequency [7].

## III. Outage Probabilities

### 3.1 Outage Probability in terms of Instantaneous SNR

Outage probability is the probability that the received SNR falls below a specified threshold value and can be expressed as:

$$P_{out}(\gamma_t) = \Pr(\gamma < \gamma_t) \quad (5)$$

where $\gamma$ is the received or instantaneous SNR and $\gamma_t$ is the specified SNR threshold which is the minimum required received SNR. Outage probability requires the knowledge of the probability density function (pdf) of the received SNR ($\gamma$), which is given by:

$$P_{out} = \int_0^{\gamma_t} p_\gamma(\gamma) d\gamma \quad (6)$$

where $p_\gamma(\gamma)$ is the pdf of $\gamma$. Assuming a Rayleigh distribution, equation (6) can be approximated as:

$$P_{out} = \int_0^{\gamma_t} \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \quad (7)$$

where $\bar{\gamma}$ is the average SNR. An outage occurs in a MIMO system when every branch is in outage; hence, the MIMO system outage probability is the product of the outage probabilities of all the branches which can be expressed as:

$$P_{out} = \prod_{i=1}^{N_s} \int_0^{\gamma_t} \frac{1}{\bar{\gamma}_i} \exp\left(-\frac{\gamma_i}{\bar{\gamma}_i}\right) \quad (8)$$

where $\gamma_i$ and $\bar{\gamma}_i$ are the received SNR and average SNR of the i-th branch, respectively, and $N_s = \min(N_R, N_T)$.

### 3.2 Outage Probability in terms of Instantaneous Capacity

A MIMO system could also be said to be in outage if the instantaneous capacity or spectral efficiency is less than the required transmission rate; and can be expressed as:

$$P_{out}(R) = \Pr\{C(H) < R\} \quad (9)$$

where $C(H)$ is the instantaneous capacity of the MIMO channel and $R$ is the required transmission rate in bit/sec/Hertz or bps/Hz. $R$ is related to the spatial multiplexing gain as:

$$R = m \log \gamma_0 \quad (10)$$

where $m$ is the spatial multiplexing gain and $\gamma_0$ is the transmitted SNR.

The instantaneous capacity of the k-th subcarrier of the OFDM over the MIMO channel, assuming $N_R \le N_T$, can be expressed as:

$$C_k(H) = \log_2 \left\{ \det\left( I_{N_R} + \hat{H}^{(k)} \hat{H}^{(k)*} \gamma_0 \right) \right\} \quad (11)$$

where * denotes the complex conjugate transpose and $I_{N_R}$ is $N_R \times N_R$ identity matrix. Thus, the total capacity of the MIMO-OFDM system is the sum of the instantaneous capacity of all the subcarriers, which is given by:

$$C(H) = \sum_{k=0}^{K-1} C_k(H) \quad (12)$$

The flow chart for calculating an outage probability in a received signal is shown in Fig. 2.

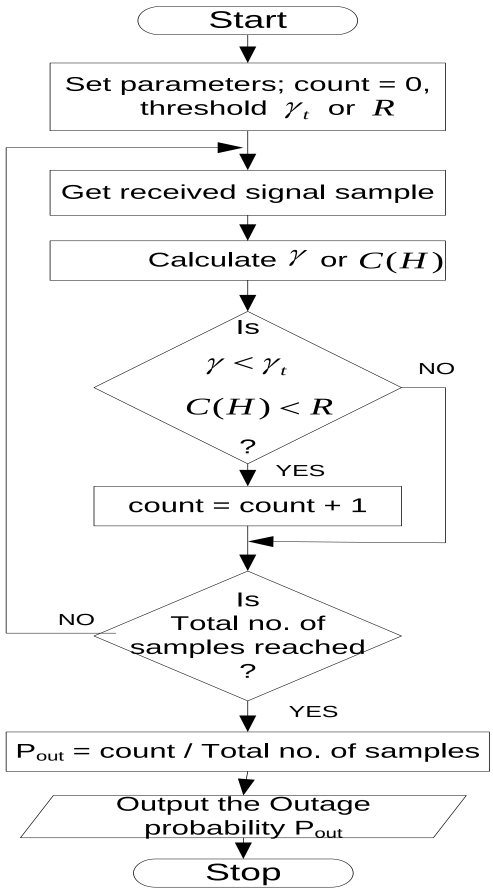


Fig. 2 Flow chart for calculating outage probability

## IV. Results and Discussion

The outage probability performance of the MIMO-OFDM system was analyzed through computer simulations in terms of both instantaneous SNR and capacity for 2x2, 3x3 and 4x4 antenna configurations. The information bits are modulated using 16-QAM and subsequently mapped to 256 subcarriers of a 256-point FFT OFDM based on IEEE802.16e standard. The CP is taken as ¼ of the OFDM symbol length. The mobile speed was set to be 120 km/h and a carrier frequency of 2.5 GHz. The Rayleigh fading channel model was used and all simulations carried out in MATLAB software environment. The performance of the MIMO-OFDM is compared with the conventional MIMO in which OFDM is not utilized.

The outage probability performances versus transmitted SNR for the MIMO-OFDM and MIMO with 2x2 antenna configuration when the SNR threshold ($\gamma_t$) is set to 5dB is presented in Fig. 3. At transmitted SNR of 10 dB, the outage probability of the MIMO-OFDM and MIMO are 0.1064 and 0.1815 respectively; this shows that the MIMO-OFDM has 41.38% reduced outage probability compared to MIMO. For all values of the transmitted SNR, the MIMO-OFDM gives lower outages compared to MIMO. Fig. 4 shows the effect of antenna configuration on the outage probability performance of the MIMO-OFDM system when $\gamma_t$ is set to 5 dB. Also, at transmitted SNR of 10 dB, the 2x3, 3x3 and 4x4 MIMO-OFDM configurations give 0.1064, 0.0722 and 0.0501 respectively. This result reveals that the system outage reduces by an average of 31% with increasing number of antennas.

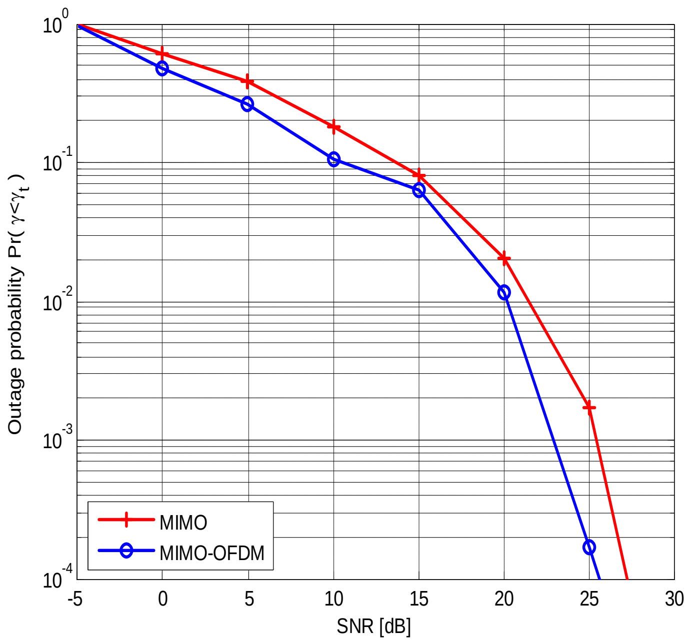


Fig. 3: Outage probability in terms of instantaneous SNR for 2x2 antenna configuration

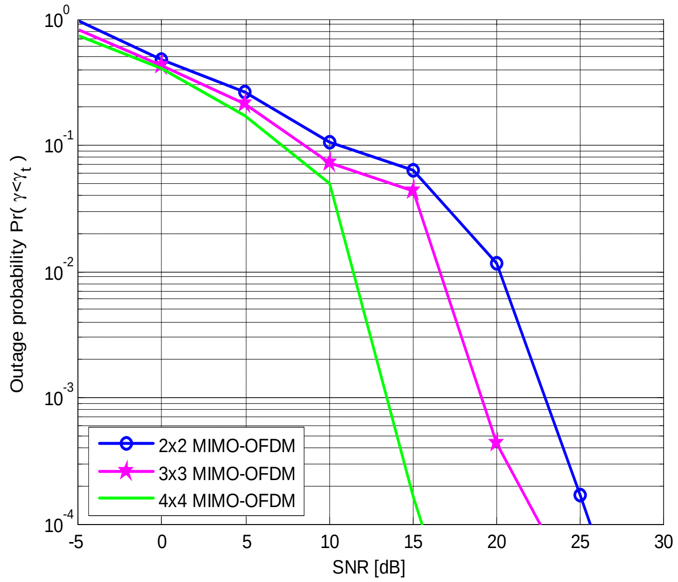


Fig. 4: Outage probability in term of instantaneous SNR for different antenna configurations

The outage probability performance in terms of instantaneous capacity was also investigated for comparison between MIMO-OFDM and MIMO. The threshold transmission rate (R) is varied from 0 to 12 bps/Hz, and the transmitted SNR at 12 dB. Fig. 5 shows the outage probabilities of both MIMO-OFDM and MIMO in terms of instantaneous capacity for 2x2 antenna configuration; taking R of 6 bps/Hz, the outage probability for MIMO-OFDM and MIMO are 0.4468 and 0.5826 respectively. This reveals that MIMO-OFDM gives 0.1358 lower outages compared to MIMO. Also from the result, the largest possible R such that the outage probability is less than 0.1 (or 10%) for MIMO-OFDM is 3.4 bps/Hz while for MIMO is 2.6 bps/Hz. This reveals that MIMO-OFDM gives higher transmission rate than MIMO for the same transmitted SNR. The MIMO-OFDM system is able to achieve this because in a fast fading environment, some of the subcarriers in a transmission branch may experience deep fading and thus low instantaneous capacities but the other subcarriers may not be in deep fade and thus experience high instantaneous capacity; thereby giving a high total instantaneous capacity. However, for the conventional MIMO, with a single carrier, an entire transmission branch could experience deep fade which results in lower total instantaneous capacity for the system.

Fig. 6 shows the outage probabilities of both MIMO-OFDM and MIMO in terms of instantaneous capacity for 3x3 antenna configuration; taking R of 6 bps/Hz, the outage probability for MIMO-OFDM and MIMO are 0.296 and 0.3966 respectively. This reveals that MIMO-OFDM gives 0.1006 lower outages compared to MIMO. Also, the largest possible R such that the outage probability is less than 0.1 (or 10%) for MIMO-OFDM is 4.2 bps/Hz while for MIMO is 3.2 bps/Hz. This also reveals that MIMO-OFDM gives higher transmission rate than MIMO for the same transmitted SNR. For the 4x4 antenna configuration, the outage probabilities of both MIMO-OFDM and MIMO in terms of instantaneous capacity are presented in Fig. 7. From the results, taking R of 6 bps/Hz, the outage probability for MIMO-OFDM and MIMO are 0.2088 and 0.3284 respectively. This reveals that MIMO-OFDM gives 0.1196 lower outages compared to MIMO. The largest possible R such that the outage probability is less than 0.1 (or 10%) for MIMO-OFDM is 5.0 bps/Hz while for MIMO is 3.8 bps/Hz. This also reveals that MIMO-OFDM gives higher transmission rate than MIMO for the same transmitted SNR.

The effect of antenna configuration on the outage probability performance of the MIMO-OFDM system in terms of instantaneous capacity is presented in Fig. 8. At transmitted SNR of 12 dB and R of 6 bps/Hz, the 2x3, 3x3 and 4x4 MIMO-OFDM configurations give 0.4468, 0.296 and 0.2088 respectively. This result reveals that the system outage reduces by an average of 32% with increasing number of antennas. This also confirms the outage probability result obtained in terms of instantaneous SNR. The results show that outage probability in terms of instantaneous capacity increases with increasing transmission rate.

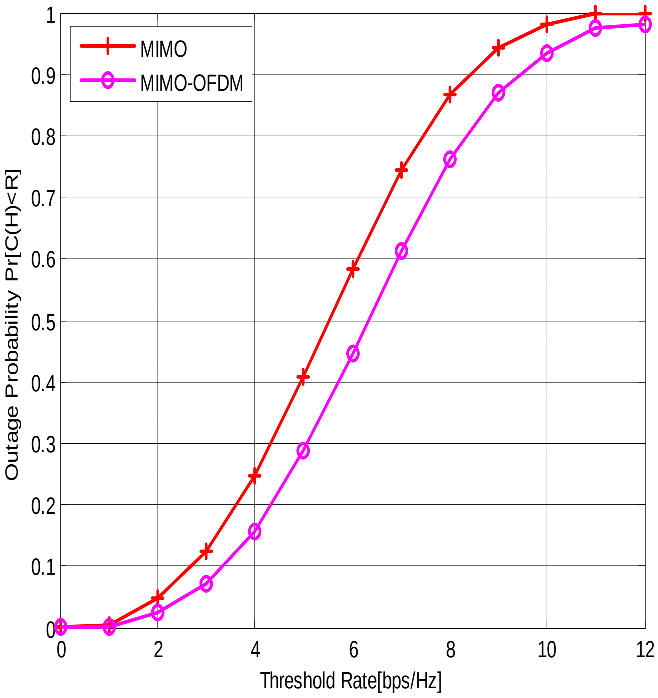


Fig. 5: Outage probability in terms of instantaneous capacity for 2x2 antenna configuration

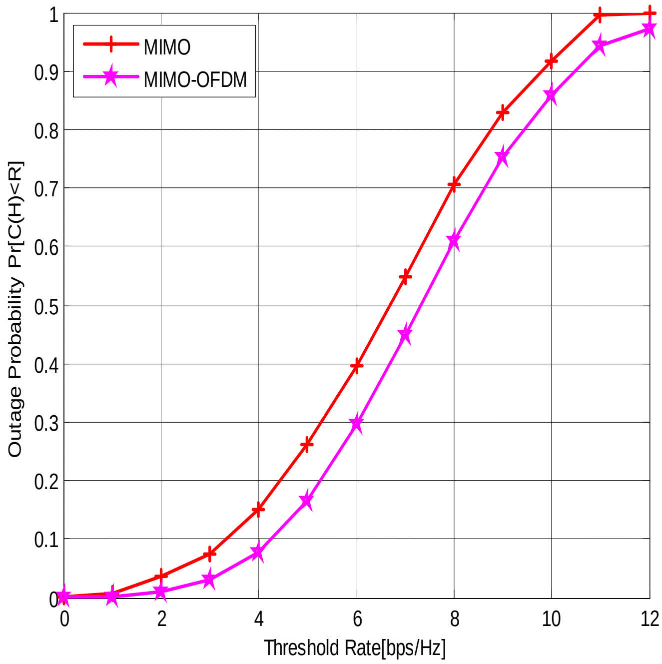


Fig. 6: Outage probability in terms of instantaneous capacity for 3x3 antenna configuration

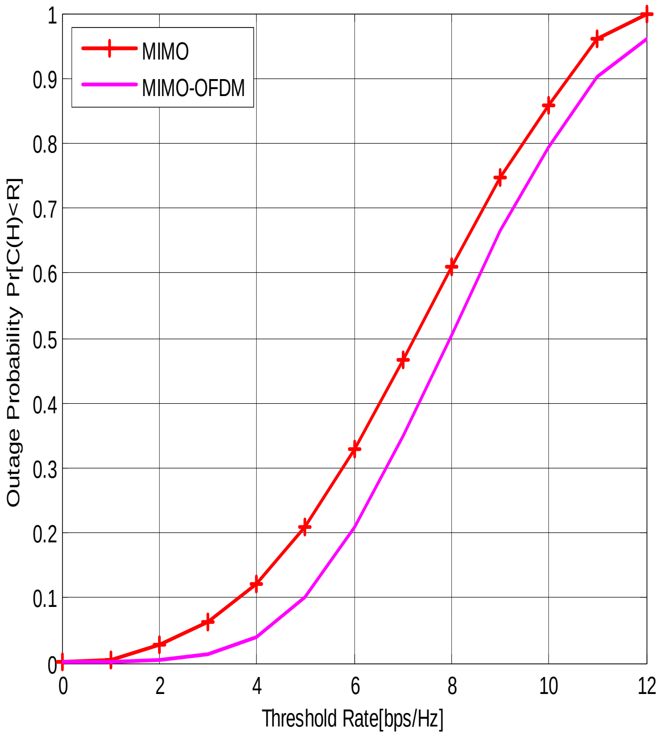


Fig. 7: Outage probability in terms of instantaneous capacity for 4x4 antenna configuration

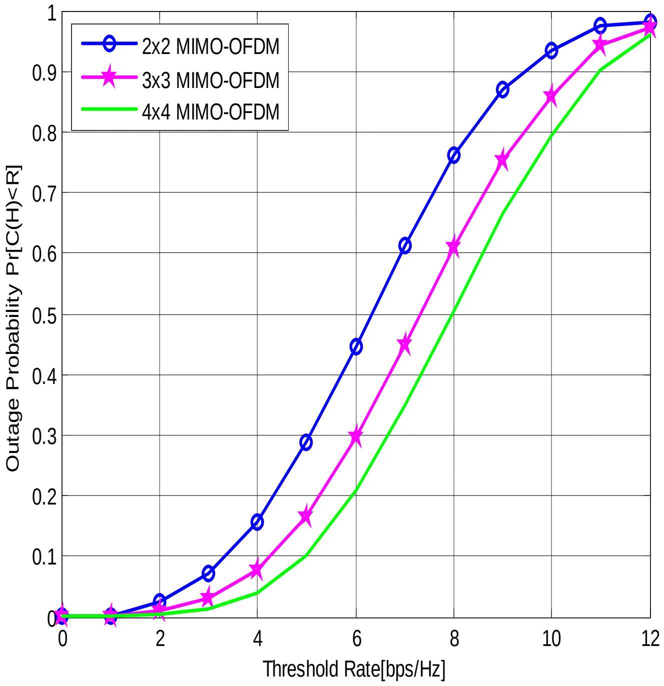


Fig. 8: Outage probability in terms of instantaneous capacity for different antenna configurations

## V. CONCLUSION

In this paper, the performance of OFDM for the reduction of outage probability in a MIMO system has been investigated. The outage probability was evaluated in terms of instantaneous SNR and instantaneous capacity. The simulation results obtained showed that the MIMO-OFDM system gives relatively lower outage probabilities compared to the conventional MIMO for all target SNRs and transmission rates.

The combination of OFDM and the MIMO system could help to eliminate the use of complex equalizers; this is because OFDM is capable of converting a fast fading channel into slow fading subchannels that can be easily handled by simple channel equalization. This also implies that the MIMO system may not require perfect CSIT. This paper has been able to show that OFDM could help to reduce the probability of signal outage in a MIMO system.

In this study, coding and interleaving were not utilized for the system so further investigations could consider incorporating coding and interleaving into the system with a view to improving the outage probability performance of the system. The use of adaptive modulation and power allocation in the system could also be investigated for very highly mobile scenarios of up to 300 km/h. The results of this paper can serve as a useful reference for researchers and designers of MIMO-OFDM system.

## REFERENCES

[1] G.J. Foschini and M.J. Gans, On limits of wireless communications in a fading environment when using multiple antennas, *Wireless Personal Communications, 6*, 1998, 311-335.

[2] I.E. Telatar, Capacity of multi-antenna Gaussian Channels, *Europ. Trans. Telecomm., 10*, 1999, 585-595.

[3] W. Mattias, *On MIMO systems and Adaptive Arrays for Wireless Communications, Analysis and Practical Aspects*, Ph.D. thesis, Signals and Systems, Uppsala University, Uppsala, Sweden, 2002.

[4] C. Mehlfuhrer, S. Caban and M. Rupp, Experimental evaluation of adaptive modulation and coding in MIMO WiMAX with limited feedback, *EURASIP Journal on Advances in Signal Processing*, 2008,

[5] T. Weber, A. Sklavos and M. Meurer, Imperfect Channel–State Information in MIMO Transmission, *IEEE transactions on communication, 54(3)*, 2006, 543-552.

[6] T.S. Rappaport *Wireless Communications – Principles and Practice*, 2nd Ed., (Prentice-Hall Inc., USA, 2002)

[7] Z.K. Adeyemo, O.O. Ajayi and F.K. Ojo, Simulation Model for a Frequency-Selective Land Mobile Satellite Communication Channel, *Innovative Systems Design and Engineering, 3( 11)*, 2012, 71-84.

[8] A. Goldsmith *Wireless Communications*, first edition, (Cambridge: University Press, Cambridge, England, 2005)

[9] Y.S. Cho, J. Kim, W.Y. Yang and C.G. Kang *MIMO-OFDM Wireless Communications with MATLAB*, (Singapore: John Wiley & Sons (Asia) Pte Ltd, 2010)

[10] O.O. Ajayi and Z.K. Adeyemo, Orthogonal Frequency Division Multiplexing for Reliable Transmission of Mobile Satellite Signals at High Data Rate, *in Proceedings of the 3rd International Conference on Mobile e-Services (ICOMeS), Ogbomoso, Nigeria, 3*, October 2011, 132-146.

[11] J. Sun, Orthogonal Frequency Division Multiplexing for Wireless Communications, *Seminar series, Wireless Communication Research Laboratory, West Virginia University*, 2003.

[12] J. Jayakumari, MIMO-OFDM for 4G Wireless Systems, *International Journal of Engineering Science and Technology*, 2(7), 2010, 2886-2889.

[13] Z.K. Adeyemo, O.O. Ajayi, R.O. Abolade, Effect of OFDM Transmission Mode on Frequency Synchronization Performance, *International Journal of Engineering*

*Research and Applications*, *3(2)*, 2013, 265-271.

[14] A.B. Awoseyila, C. Kasparis and B.G. Evans, Robust time-domain timing & frequency synchronization for OFDM systems, *IEEE Transactions on Consumer Electronics*, *55(2)*, 2009, 391-399.

[15] H. Zhao, Y. Liu, J. Zhang and J. Zhou, A novel joint synchronization algorithm for OFDM systems based on single training symbol, *International Journal of Digital Content Technology and its Applications*, *5(5)*, 2011, 217-225.

[16] J. Huang and S. Signell, Asymptotic Performance of MMSE MIMO Systems Using Adaptive Modulation, *IEEE Communication Letter,* October, 2008.